AP Calculus - 2012-09-18

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

21) $\lim_{x\to 0^+} \frac{\sin x}{5\sqrt{x}}$

$= \lim_{x\to 0^+} \left(\frac{x}{5\sqrt{x}}\right)\left(\frac{\sin x}{x}\right) = \lim_{x\to 0^+} \left(\frac{\sqrt{x}}{5}\right)\left(\frac{\sin x}{x}\right)$

$= 0$

$\frac{x}{\sqrt{x}} = \frac{x^1}{x^{1/2}} = x^{1-\frac{1}{2}} = x^{\frac{1}{2}}$

$\| $

$\frac{(\sqrt{x})^2}{\sqrt{x}} = \sqrt{x}$ *

*if $x \geq 0$

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2.6/27

$$\lim_{\theta\to 0}\frac{\theta^2}{1-\cos\theta}$$

$$\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta}=0$$

$$\sin^2\theta+\cos^2\theta=1$$
$$\sin^2\theta=1-\cos^2\theta$$

$$=\lim_{\theta\to 0}\frac{\theta}{1}\left(\frac{\theta}{1-\cos\theta}\right)\left(\frac{1+\cos\theta}{1+\cos\theta}\right)$$

$$=\lim_{\theta\to 0}\frac{\theta(1+\cos\theta)}{1}\left(\frac{\theta}{1-\cos^2\theta}\right)$$

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$$

$$\lim_{\theta\to 0}\frac{\theta}{\sin\theta}=\frac{1}{1}=1$$

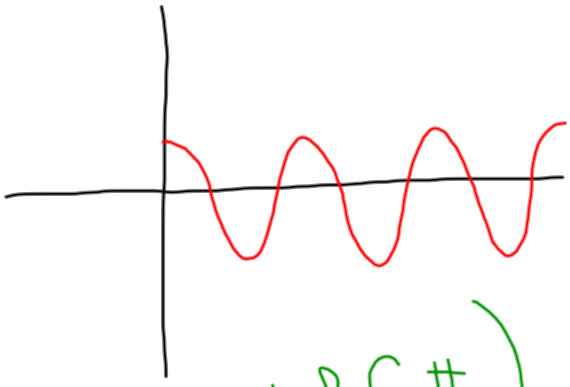
$$=\lim_{\theta\to 0}\frac{\theta(1+\cos\theta)}{\sin\theta}\left(\frac{\theta}{\sin\theta}\right)$$

$$=\lim_{\theta\to 0}(1+\cos\theta)\left(\frac{\theta}{\sin\theta}\right)^2=(2)(1)^2$$

$$=2$$

29) $\lim_{\theta \to 0} \frac{\theta}{\cos\theta} = \frac{0}{1} = 0$

33) $\lim_{x \to 0^+} \cos\left(\frac{1}{x}\right) =$



DNE (b/c $\cos(x)$ oscillates back & forth $x \to \infty$)

31) $\lim_{\theta\to 0} \frac{1-\cos 5\theta}{\cos 7\theta - 1} =$

$\lim_{x\to 0} \frac{1-\cos x}{x} = 0$

$\sin^2\theta+\cos^2\theta=1$

$\sin^2\theta+\cos^2\theta-1=0$

$$\lim_{\theta\to 0} \frac{(1-\cos 5\theta)}{(\cos 7\theta - 1)} \cdot \frac{(\cos 7\theta+1)}{(\cos 7\theta+1)} = \lim_{\theta\to 0} \left(\frac{1-\cos 5\theta}{1}\right)\left(\frac{\cos(7\theta)+1}{\cos^2 7\theta - 1}\right)$$

$$= \lim_{\theta\to 0} (1-\cos 5\theta)(1+\cos 7\theta)\left(\frac{-1}{\sin^2 7\theta}\right)\left(\frac{(7\theta)^2}{(7\theta)^2}\right)$$

$$= \lim_{\theta\to 0} \left(\frac{\cos 5\theta - 1}{49\theta^2}\right)(1+\cos 7\theta)\left(\frac{7\theta}{\sin 7\theta}\right)^2$$

$$= \lim_{\theta\to 0} \left(\frac{\cos 5\theta - 1}{49\theta^2}\right)\left(\frac{\cos 5\theta+1}{\cos 5\theta+1}\right)(1+\cos 7\theta)\left(\frac{7\theta}{\sin 7\theta}\right)^2$$

$$= \lim_{\theta\to 0} \frac{\cos^2 5\theta - 1}{49\theta^2}\left(\frac{1+\cos 7\theta}{\cos 5\theta+1}\right)\left(\frac{7\theta}{\sin 7\theta}\right)^2$$

$$= \lim_{\theta\to 0} \frac{-\sin^2 5\theta}{49\theta^2}\left(\frac{1+\cos 7\theta}{\cos 5\theta+1}\right)\left(\frac{7\theta}{\sin 7\theta}\right)^2$$

$$= \lim_{\theta\to 0} -\left(\frac{1+\cos 7\theta}{\cos 5\theta+1}\right)\left(\frac{\sin^2 5\theta}{(5\theta)^2}\right)\left(\frac{25}{49}\right)\left(\frac{7\theta}{\sin 7\theta}\right)^2$$

$-\frac{25}{49}$ $-\frac{2}{2}$ 1^2 $\left(\frac{\sin 5\theta}{5\theta}\right)^2$ $\frac{25}{49}$ 1^2

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2.6 #23

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$$\lim_{x\to 0} \frac{\tan 7\theta}{\sin 3\theta} = \lim_{x\to 0} \frac{\sin 7\theta}{\cos 7\theta} \cdot \frac{1}{\sin 3\theta}$$

$$= \lim_{x\to 0} \left(\frac{1}{\cos 7\theta}\right)\left(\frac{\sin 7\theta}{7\theta}\right)\left(\frac{3\theta}{\sin 3\theta}\right)\left(\frac{7}{3}\right) = \frac{7}{3}$$

$\frac{1}{\cos 7\theta} \to 1$

$\frac{\sin 7\theta}{7\theta} \to 1$

$\frac{3\theta}{\sin 3\theta} \to 1 \div \left(\frac{1}{1}\right)$

$\frac{7}{3}$

$$\frac{\tan 7\theta}{\sin 3\theta} = \tan 7\theta \left(\frac{1}{\sin 3\theta}\right)$$